Mathematical and Theoretical Foundations of AI

Module no. or code	1
Module name	Mathematical and Theoretical Foundations of AI
(If applicable) the module's courses	
Module content	1. Advanced Vector Calculus • Multivariate derivatives and chain rule • Backpropagation and automatic differentiation • Linearization and multivariate Taylor series 2. Advanced Linear Algebra • Eigenvalues and eigenvectors • Singular value decomposition • Matrix approximation 3. Continuous Optimization • Gradient descent • Constrained optimization and Lagrange multipliers • Convex Optimization 4. Models and Data • Change of variables • Empirical risk minimization • Parameter estimation • Probabilistic modelling and inference • Model selection 5. Basic Applications for AI systems • Linear Regression • Dimensionality Reduction with Principal Component Analysis (PCA) • Density Estimation with Gaussian Mixture Models

<table>
<tr><td>Module’s learning outcomes</td><td>- Students refresh and develop further their knowledge and skills on the necessary mathematical foundations for understanding and developing algorithms for AI; in particular, linear algebra, calculus, probability.
- Students understand the principles of continuous optimization (constrained and unconstrained), are able to select appropriate approaches and they apply them for problems in AI.
- Students are able to apply and evaluate the principles of probabilistic modelling and inference, and they create probabilistic models for frequently occurring kinds of data.
- Students use the acquired mathematical skills to design and create frequently occurring building blocks of AI systems, such as linear regression, PCA, Gaussian mixture models and support vector machines.</td></tr>
<tr><td>Semester</td><td>1st semester</td></tr>
<tr><td>Duration of module</td><td>One semester</td></tr>
<tr><td>Frequency</td><td>Summer term only</td></tr>
<tr><td>ECTS-Credits</td><td>5</td></tr>
<tr><td>Workload</td><td><table><tr><td>Workload (Total)</td><td>Attendance time</td><td>Self-Study time (incl. exam preparation)</td></tr><tr><td>150</td><td>60</td><td>90</td></tr></table></td></tr>
<tr><td>Type of module</td><td>Compulsory</td></tr>
<tr><td>Applicability of module</td><td>Foundation to multiple modules of the second semester. In parts the module is required side a side and consecutive for other modules in the first semester.</td></tr>
<tr><td>Conditions for participation</td><td></td></tr>
<tr><td>Responsible for module</td><td>Prof. Dr. Martin Storath</td></tr>
<tr><td>Lecturer</td><td>Prof. Dr. Martin Storath, Prof. Dr. Kai Diethelm</td></tr>
<tr><td>Language of instruction, L. of examination</td><td>english</td></tr>
<tr><td>Type of examination;
Conditions for the award of CPs</td><td>Written exam</td></tr>
<tr><td>Teaching and learning formats of the module</td><td>seminar-based teaching</td></tr>
</table>

Literature	1. M. P. Deisenroth, A. A. Faisal, Cheng Soon Ong: *Mathematics for Machine Learning,* Cambridge University Press, 2020 2. *C. M. Bishop: Pattern Recognition and Machine Learning, Springer, 2006* 3. G. James, D. Witten, T. Hastie, R. Tibshirani: *An Introduction to Statistical Learning,* Second Edition, Springer, 2021